1) A jet lands on an aircraft carrier at 140 mi/h .

(a) What is its acceleration if it stops in 2.0 s?

$$v_{xf} = v_{xi} + a_x t$$
$$x_f - x_i = \tfrac{1}{2}(v_{xi} + v_{xf})\,t$$
$$x_f - x_i = v_{xi}t + \tfrac{1}{2}a_x t^2$$
$$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$$

$$a_x = \frac{v_{xf} - v_{xi}}{t} \approx \frac{0 - 63\ \text{m/s}}{2.0\ \text{s}} = -31\ \text{m/s}^2$$

(b) What is the displacement of the plane while it is stopping?

$$x_f - x_i = \tfrac{1}{2}(v_{xi} + v_{xf})\,t = \tfrac{1}{2}(63\ \text{m/s} + 0)(2.0\ \text{s}) = 63\ \text{m}$$

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2) A car accelerates down a straight road at the rate of 1.8 m s^{-2} . it take the care 14 s to travel the distance between two marker flags that are 300m apart. How fast was the car moving when it passed the first flag?

$x = v_0\,t + 1/2\ a\,t^2$
$v_0\,t = 300 - 1/2\ (\ 1.8\ (14)^2)$
$14\ v_0 = 123.6$
$v_0 = 8.83$ m/s

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3) From a standing start, a skier (متزحلق) accelerates down a long straight hill. She travels a distance of 120 m with a constant acceleration of 0.8 m s^{-2} . how long did it take her to cover that distance?

$x = v_0\,t + 1/2\ a\,t^2$
$t^2 = 2\ x/a$
$t^2 = (\ 2 \times 120\ ) / 0.8 = 300$
$t = 17.3$ s

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4) The speed for small racing airplane increase from 60 m s^{-1} to 75 m s^{-1} in 3.5 s. determine the acceleration of the plane during this interval

$v_f = v_0 + a\,t$
$a = (v_f - v_0) / t$
$a = 4.29$ m / s^2

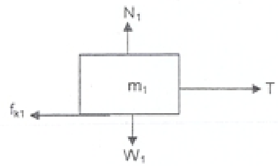
Two blocks connected by a light rope are being dragged by a horizontal force **F**(see figure). Suppose that F=50 N and m_1=10 kg, m_2=20kg and The coefficient of kinetic friction between each block and the surface is 0.1, find:

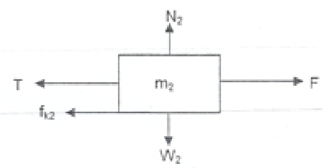
a) The frictional force acting on the block m_2?

b) The acceleration of the system

c) The tension T?

d) The speed of the system after it has slid **1** m if start from rest?





(a) $\Sigma F_x = ma$ $\quad T - f_{k1} = m_1 a$ (1) $\qquad \Sigma F_x = ma$ $\quad F - T - f_{k2} = m_2 a$ (2)

$\Sigma F_y = ma$ $\quad N_1 - W_1 = 0$ $\qquad \Sigma F_y = ma$ $\quad N_2 - W_2 = 0$

$N_1 = m_1 g = (10)(9.8) = 98$ N $\qquad N_2 = m_2 g = (20)(9.8) = 196$ N

$f_{k1} = \mu_k N_1 = (0.1)(98) = 9.8$ N $\qquad f_{k2} = \mu_k N_2 = (0.1)(196) = 19.6$ N

(b) Adding eq.(1) & (2)

$F - T - f_{k2} + T - f_{k1} = m_1 a + m_2 a$ $\qquad F - f_{k2} - f_{k1} = (m_1 + m_2)a$

$$a = \frac{F - f_{k2} - f_{k1}}{m_1 + m_2} = \frac{50 - 19.6 - 9.8}{30} = 0.687\,m/s^2$$

(c) $T = f_{k1} + m_1 a = 9.8 + 10(0.687) = 16.7$ N

(d) $v^2 = v_0^2 + 2ax$ $\qquad \rightarrow v = \sqrt{2ax} = \sqrt{2(0.687)(1)} = 1.17\,m$

Exercise:

Given $\bar{a}$ and $\bar{b}$ as shown in the diagram.
Find:

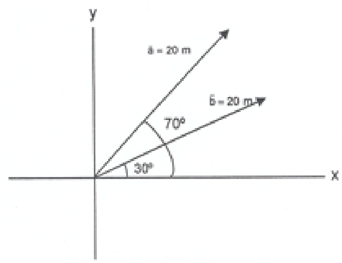
a) $\bar{a}+\bar{b}$

b) $\bar{a}\times\bar{b}$ $\quad |\bar{a}\times\bar{b}|$

c) $\bar{a}\cdot\bar{b}$

d) $\bar{a}-2\bar{b}$

e) What is the angle between vector $\bar{a}$ and vector $\bar{b}$



$a_x = a\cos 30 = 20\cos 30 = 17.32\text{ m}$, $a_y = a\sin 30 = 20\sin 30 = 10\text{ m}$

$\bar{a} = 17.32\hat{i} + 10\hat{j}$

$b_x = b\cos\theta = 20\cos 70 = 6.84$, $b_y = b\sin\theta = 20\text{Sin}70 = 18.79$

$\bar{b} = 6.84\hat{i} + 18.79\hat{j}$

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a)

$$\bar{a}+\bar{b} = 17.32\hat{i} + 10\hat{j} + 6.84\hat{i} + 18.79\hat{j} = 24.16\hat{i} + 28.79\hat{j}$$

$$\bar{c} = 24.16\hat{i} + 28.79\hat{j}$$

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b)

$$\bar{a}\times\bar{b} = \begin{vmatrix} 17.32 & 10 & 0 \\ 6.84 & 18.79 & 0 \end{vmatrix} = [(10)(0)-(0)(18.79)]\hat{i} + [(0)(6.84)-(0)(17.32)]\hat{j} + [(17.32)(18.79)-(10)(6.84)]\hat{k}$$

$$\bar{d} = (325.4 - 68.84)\hat{k} = 257\hat{k}$$

$$|\bar{a}\times\bar{b}| = \sqrt{(257)^2} = 66049$$

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c) $\quad \bar{a}\cdot\bar{b} = (17.32\hat{i} + 10\hat{j})\cdot(6.84\hat{i} + 18.79\hat{j}) = 118.47 + 187.9 = 306.4\text{m}$

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d) $\quad \bar{a} - 2\bar{b} = (17.32\hat{i} + 10\hat{j}) - 2[(6.84\hat{i} + 18.79]\hat{j} = 3.6\hat{i} - 27.58\hat{j}$

e) $\quad \theta = 40°$

Q.1 Given three vectors $\vec{a} = 6\hat{i} - 2\hat{j} + 4\hat{k}$, $\vec{b} = \hat{i} + \hat{j} - 4\hat{k}$ and $\vec{c} = 6\hat{i} + 6\hat{j} + 2\hat{k}$ then Find

a) $4\vec{a} - 8\vec{b} + \vec{c}$

b) $\vec{a} - \vec{b} + 2\vec{c}$

c) The magnitude of $\vec{a} + 4\vec{b} - 3\vec{c}$

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a)

$4\vec{a} - 8\vec{b} + \vec{c} = 4(6\hat{i} - 2\hat{j} + 4\hat{k}) - 8(\hat{i} + \hat{j} - 4\hat{k}) + (6\hat{i} + 6\hat{j} + 2\hat{k}) = 24\hat{i} - 8\hat{j} + 16\hat{k} - 8\hat{i} - 8\hat{j} + 32\hat{k} + 6\hat{i} + 6\hat{j} + 2\hat{k}$

$4\vec{a} - 8\vec{b} + \vec{c} = 22\hat{i} - 10\hat{j} + 50\hat{k}$

b)

$\vec{a} - \vec{b} + 2\vec{c} = (6\hat{i} - 2\hat{j} + 4k) - (\hat{i} + \hat{j} - 4\hat{k}) + 2(6\hat{i} + 6\hat{j} + 2\hat{k}) = 6\hat{i} - 2\hat{j} + 4\hat{k} - \hat{i} - \hat{j} + 4\hat{k} + 12\hat{i} + 12\hat{j} + 4\hat{k}$

$\vec{a} - \vec{b} + 2\vec{c} = 17\hat{i} + 9\hat{j} + 12k$

c)

$\vec{a} + 4\vec{b} - 3\vec{c} = (6\hat{i} - 2\hat{j} + 4\hat{k}) + 4(\hat{i} + \hat{j} - 4\hat{k}) - 3(6\hat{i} + 6\hat{j} + 2\hat{k}) = 6\hat{i} - 2\hat{j} + 4\hat{k} + 4\hat{i} + 4\hat{j} - 16\hat{k} - 18\hat{i} - 18\hat{j} - 6\hat{k}$

$\vec{a} + 4\vec{b} - 3\vec{c} = -8\hat{i} - 16\hat{j} - 18\hat{k}$

$\vec{F} = \sqrt{(-8)^2 + (-16)^2 + (-18)^2} = \sqrt{64 + 256 + 324} = \sqrt{644} = 25.4$

Q.2 If $\vec{a} + \vec{b} = 2\vec{c}$, $\vec{a} - \vec{b} = 4\vec{c}$ and $\vec{c} = 3\hat{i} + 4\hat{j}$ then what are $\vec{a}$ and $\vec{b}$?

$$\vec{a} + \vec{b} = 2\vec{c}$$
$$\vec{a} - \vec{b} = 4\vec{c}$$
$$2\vec{a} = 6\vec{c}$$

$$\dashrightarrow \quad \vec{a} = 3\vec{c}$$

$$\vec{a} = 3\vec{c} = 3(3\hat{i} + 4\hat{j}) = 9\hat{i} + 12\hat{j}$$

$$\vec{a} + \vec{b} = 2\vec{c} = 2(3\hat{i} + 4\hat{j})$$

$$9\hat{i} + 12\hat{j} + \vec{b} = 6\hat{i} + 8\hat{j}$$

$$\vec{b} = 6\hat{i} + 8\hat{j} - 9\hat{i} - 12\hat{j} = -3\hat{i} - 4\hat{j}$$

Given a Vector $\bar{A} = 5\hat{i} + 2\hat{j} - 2\hat{k}$, find:

a) the angle betwween $\bar{A}$ and each axis.

The magnitude of the vector is

$$|\bar{A}| = \sqrt{(5)^2 + (2)^2 + (-2)^2} = \sqrt{33} = 5.74$$

Therefore the angle that vector $\bar{A}$ makes with x-axis is

$$\theta_x = \cos^{-1}\left[\frac{A_x}{|\bar{A}|}\right] = \cos^{-1}\left[\frac{5}{5.74}\right] = \cos^{-1}(0.871) = 29.4°$$

The angle that vector $\bar{A}$ makes with y-axis is

$$\theta_y = \cos^{-1}\left[\frac{A_y}{|\bar{A}|}\right] = \cos^{-1}\left[\frac{2}{5.74}\right] = \cos^{-1}(0.348) = 69.6°$$

The angle that vector $\bar{A}$ makes with z-axis is

$$\theta_z = \cos^{-1}\left[\frac{A_z}{|\bar{A}|}\right] = \cos^{-1}\left[\frac{-2}{5.74}\right] = \cos^{-1}(-0.348) = 110.4°$$

b) the angle betwween $\bar{A}$ and negative x-axis is .

A particle moves along the x-axis. Its x coordinate varies with time according to the expression $x = -4t + 2t^2$, where x is in meters and t in seconds.
(a) Determine the displacement of the particle in time intervals t=0 to t=1 s and t=1 and t=3 s.
(b) Calculate the average velocity in the time intervals t=0 to t=1 s and t=1 s to t=3 s
(c) Find the instantaneous velocity of the particle at t=2.5 s.
(d) The instantaneous acceleration of the particle at t=3.0 s.

(a) $\Delta x_{01} = x(1s) - x(0) = [-4(1) + 2(1)^2] - [-4(0) + 2(0)^2] = -2m$

$\Delta x_{13} = x(3s) - x(1) = [-4(3) + 2(3)^2] - [-4(1) + 2(1)^2] = [6] - [-2] = 8\ m$

(b) The average velocity in the time intervals t=0 to t=1 s

$$\bar{v}_{01} = \frac{x(1s) - x(0)}{\Delta t} = \frac{\Delta x_{01}}{\Delta t} = \frac{-2}{1} = -2\ m/s$$

$$\bar{v}_{13} = \frac{x(3s) - x(1)}{\Delta t} = \frac{\Delta x_{13}}{\Delta t} = \frac{8}{2} = 4\ m/s$$

(c) The instantaneous velocity of the particle at time t is given by:

$$v = \frac{dx}{dt} = -4 + 4t\ m/s \quad ,\text{at } t = 2.5s \quad v = -4 + 4(2.5) = 6\ m/s$$

(d) The instantaneous acceleration of the particle at time t is given by:

$$a = \frac{dv}{dt} = 4\ m/s^2$$

It is constant, so the acceleration at any time is 4 m/s^2

The position $\vec{r}$ of a particle moving in an xy plane is given by $\vec{r} = (2t^3 - 5t)\hat{i} + (6 - 7t^4)\hat{j}$. Here $\vec{r}$ is in meters and t in seconds. What are the magnitude and direction of the particle's position vector $\vec{r}$, particle's velocity vector $\vec{v}$ and particle's acceleration $\vec{a}$ at t=2 Sce.

At=2 sec.

$$\vec{r} = \left[2(2)^2 - 5(2)\right]\hat{i} + \left[6 - 7(2)^4\right]\hat{j} \ \ m \qquad \vec{r} = (16 - 10)\hat{i} + (6 - 112)\hat{j} \ \ m$$

$$\vec{r} = (6\hat{i} - 106\hat{j}) \ \ m$$

$$\vec{r} = \sqrt{r_x^2 + r_y^2} \ \sqrt{(6)^2 + (-106)^2} = 106.2m, \qquad \theta = \tan^{-1}(\frac{r_y}{r_x}) = \tan^{-1}(\frac{-106}{6}) = -86.8°$$

θ=273.2°

$$\vec{V} = \frac{d\vec{r}}{dt} = (6t^2 - 5)\hat{i} + (-28t^3)\hat{j}$$

At t=2 Sec.

$$\vec{V} = \left[6(2)^2 - 5\right]\hat{i} + \left[-28(2)^3\right]\hat{j} = (24 - 5)\hat{i} + (-224)\hat{j} = (19\hat{i} - 224\hat{j}) \ \ m/s$$

$$\vec{v} = \sqrt{v_x^2 + v_y^2} = \sqrt{(19)^2 + (-224)^2} = 224 \ \ m/s$$

$$\theta = \tan^{-1}(\frac{V_y}{V_x}) = \tan^{-1}(\frac{-224}{19}) = -85° \qquad \text{OR} \quad \theta = 275°$$

$$\vec{a} = \frac{dv}{dt} = (12t)\hat{i} + (-84t^2)\hat{j}$$

At t=2 Sec.

$$\vec{a} = \left[12(2)\right]\hat{i} + \left[-84(2)^2\right]\hat{j} = (24\hat{i} - 336\hat{j}) \ \ m/s^2$$

$$\vec{a} = \sqrt{a_x^2 + a_y^2} = \sqrt{(24)^2 + (-336)^2} = 336.9 \ \ m/s^2$$

$$\theta = \tan^{-1}(\frac{a_y}{a_x}) = \tan^{-1}(\frac{-336.9}{24}) = -86° \qquad \text{OR} \quad \theta = 274°$$

Suppose that it takes 10 h to drain a container of 5700 m^3 of water. What is the "mass flow rate " in kilograms per second, of water from the contaner.

Time =10 h=(10)(60)(60)=36000 s

$$\rho = \frac{m}{V}$$

The Volume= 5700 m^3

$$\rho = \frac{m}{V} \quad \text{------------} \rightarrow m = \rho V = (1000)(5700) = 5.7 \times 10^6 \text{ kg}$$

$$\text{Mass flow rate} = \frac{\text{mass}}{\text{Time}} = \frac{(5.7 \times 10^6)}{(3.6 \times 10^4)} = 158.33 \text{ kg/s}$$